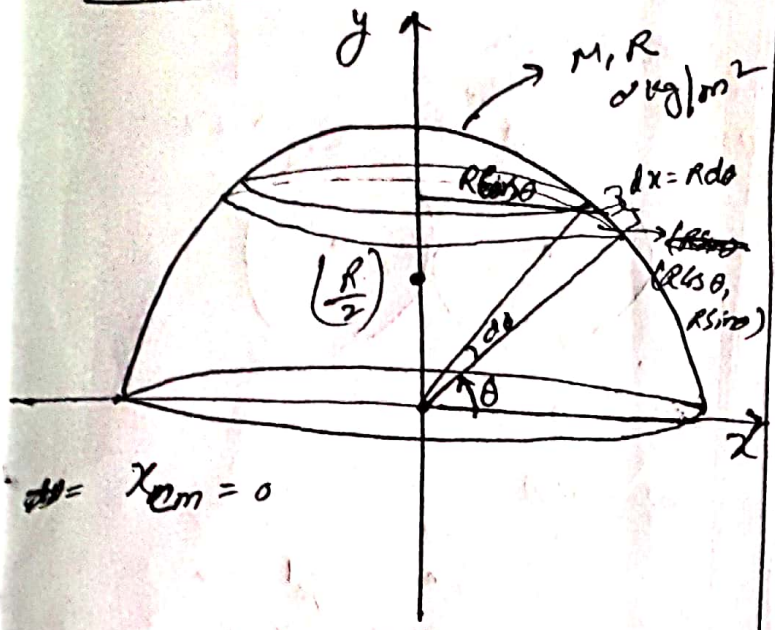
⇒ COM of Hollow Hemi-sphere :-



$x_{cm} = 0$

$dm = \sigma \cdot 2\pi R\cos\theta \cdot R\,d\theta$

$dm = \sigma \cdot 2\pi R^2 \cos\theta \, d\theta$

$$y_{cm} = \frac{\int_0^{\pi/2} \sigma \cdot 2\pi R^2 \cos\theta \cdot d\theta \cdot R\sin\theta}{\int_0^{\pi/2} \sigma \cdot 2\pi R^2 \cos\theta \cdot d\theta}$$

$$y_{cm} = \frac{R}{2} \cdot \frac{\int_0^{\pi/2} \sin\theta \cos\theta \, d\theta}{[\sin\theta]_0^{\pi/2}}$$

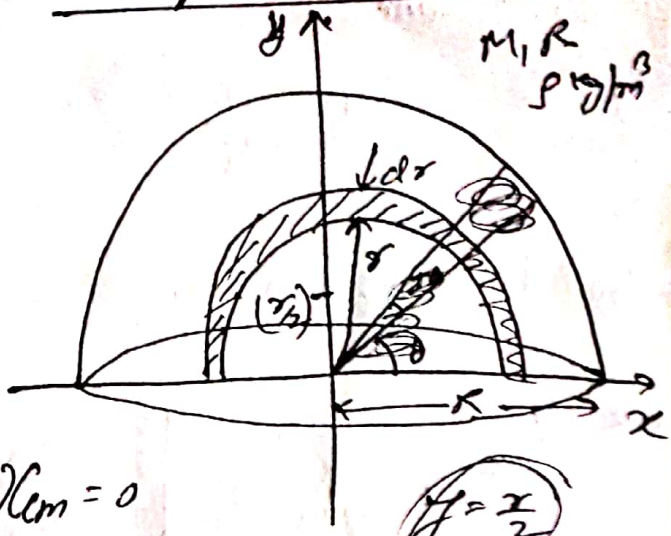
$$y_{cm} = \frac{R}{2} \cdot \left\{ -\frac{\cos 2\theta}{2} \right\}_0^{\pi/2}$$

$$y_{cm} = \frac{R}{4} \left\{ -\cos\cdot 2\cdot\pi/2 - (-\cos 0) \right\}$$

$$y_{cm} = \frac{2R}{4} = \frac{R}{2}$$

$$x_{cm}, y_{cm} = \left(0, \frac{R}{2}\right)$$

⇒ COM of Solid Hemisphere



$x_{cm} = 0$

$y = \frac{r}{2}$

$dm = \rho \cdot dV$

$= \rho \cdot 2\pi r^2 \cdot dr$

$$y_{cm} = \frac{\int_0^R dm \cdot y}{\int_0^R dm}$$

$$y_{cm} = \frac{\int_0^R \rho \cdot 2\pi r^2 dr \cdot \frac{r}{2}}{\int_0^R \rho \cdot 2\pi r^2 dr}$$

$$y_{cm} = \frac{1}{2} \cdot \frac{\int_0^R r^3 \cdot dr}{\int_0^R r^2 dr}$$

$$y_{cm} = \frac{1}{2} \cdot \frac{\left[\frac{R^4}{4}\right]}{\left[\frac{R^3}{3}\right]}$$

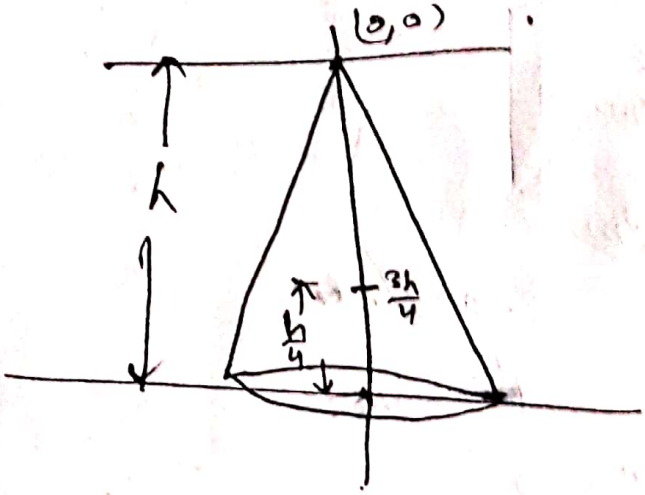
$$y_{cm} = \frac{3}{8} R$$

$$x_{cm}, y_{cm} = 0, \frac{3}{8} R$$

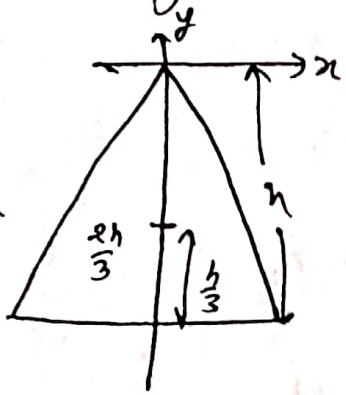
Solid Cone

from top :- $y = \frac{3h}{4}$

from bottom, $y_{cm} = \frac{h}{4}$



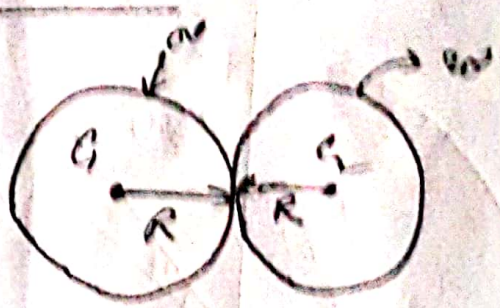
Triangular Plate :-



$y_{cm} = \frac{2h}{3}$

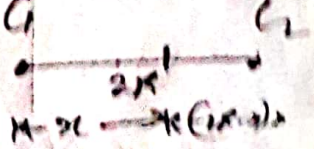
Com of Combination of Structures :-

Disc



$m_1 = \sigma A$

$m_2 = \sigma \cdot A$

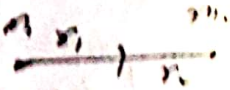


$\sigma A \cdot x = \sigma A (2R - x) \cdot \sigma$

$x = 2R - 2x$

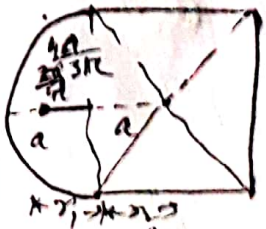
$3x = 4R$

$x = \frac{4R}{3}$



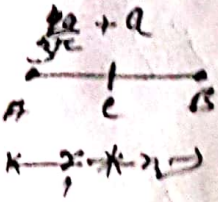
$x_1 = \frac{m_2 \cdot x_2}{m_1 + m_2}$

#



$m_1 = \sigma \times \frac{\pi a^2}{2}$

$m_2 = \sigma \times 4a^2$



$\sigma \times \frac{\pi a^2}{2} \times x_1 = \sigma \times 4a^2 \times x_2$

$\frac{\pi}{2} \cdot x_1 = 4 \cdot x_2$

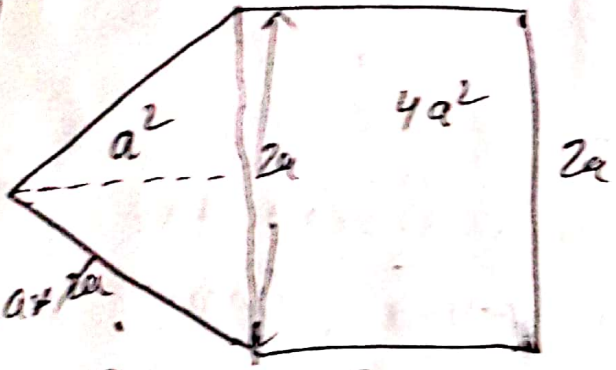
$x_1 = \frac{8}{\pi} x_2$

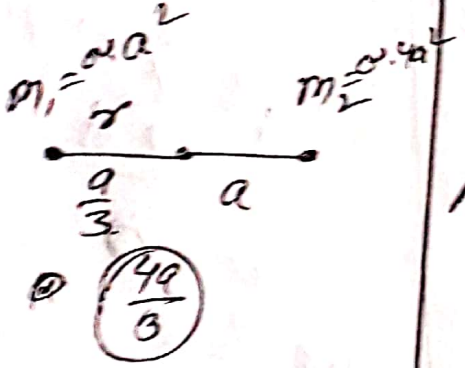
$$\sigma \cdot \frac{\pi a^2}{2} \times x = 2\sigma_0 \cdot 4a^2 \left[\frac{4a}{3\pi} + a - x\right]$$

$$\pi x = 16\left[\frac{4a + 3\pi a}{3} - x\right]$$

$\pi x = ?$

find com.





$\frac{4a}{3}$

$$\vec{r}_1 = \frac{m_2 r}{m_1 + m_2}$$

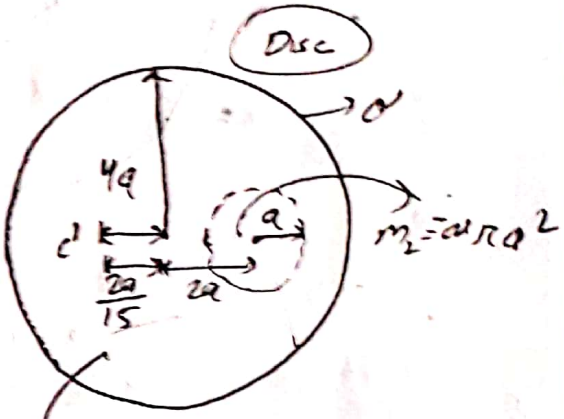
$$\vec{r}_1 = \frac{\sigma \times 4a^2 \times \left(\frac{4a}{3}\right)}{5a^2 \sigma}$$

$$r = \frac{16}{15} a$$

Cavity Problems

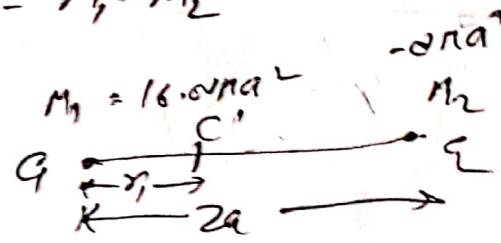
Cavity means ek aisi khali jagah jo kisi baaf se band ho.

#



$M_1 = \sigma \pi \cdot 16a^2$

Remaining $= M_1 - M_2$



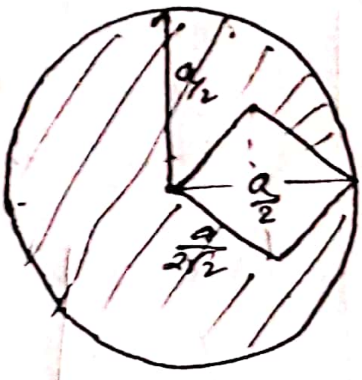
$$r_1 = \frac{-\sigma \pi a^2 \times 2a}{15\sigma_0 \cdot \pi a^2}$$

$$r_1 = -\frac{2a}{15} \pi$$

A square hole is punched out of a circular lemina. diagnol of this square is the circle's radius. If a be dia of the circle then find the COM from the circle's centre.

$M_1 = \frac{\sigma \pi a^2}{4}$

$M_2 = \frac{\sigma \cdot a^2}{8}$



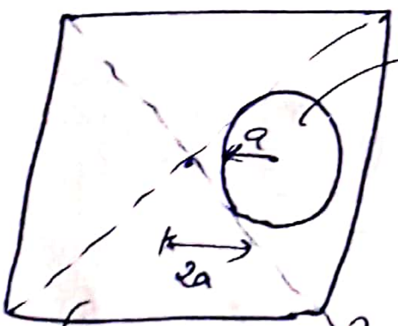
$r_1 = \frac{-m_2 \cdot r}{m_1 + m_2}$

$\frac{\sigma \pi a^2}{4}$ C $\frac{a}{4}$ $\frac{-\sigma a^2}{8}$, r_1

$$r_1 = -\frac{\frac{\sigma a^2}{8} \cdot \frac{a}{4}}{\frac{\sigma \pi a^2}{4} - \frac{\sigma a^2}{8}}$$

$$r_1 = \frac{-a}{4(2\pi - 1)}$$

#



$M_2 = \sigma \cdot \pi a^2$

$M_1 = \sigma (8a)^2$
$= 64 \sigma a^2$

$r - a = \frac{a}{2}$

$64\sigma a^2$ C $2a$ $-\sigma \pi a^2$, r

$$r_1 = \frac{-\sigma \pi a^2 \cdot 2a}{64 \sigma a^2 - \sigma \pi a^2}$$

$$r_1 = \frac{-2\pi a}{64 - \pi}$$

Motion of COM and Conservation of linear momentum:-

$\vec{F}_{net} = 0$

$m\vec{a}_{cm} = 0$

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots + m_n \vec{r}_n}{m_1 + m_2 + \cdots + m_n}$$

$\vec{V}_{cm} = \frac{m_1 \vec{r}}{2}$

$\vec{V}_{cm} = m_1 \cdot v_1 + u \cdot r t$